α - DESIGNS

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1. Introduction

Block designs are useful in experiments involving one-way elimination of heterogeneity. A block design is said to be incomplete if it contains at least one block that does not have full set of treatments. The commonly used block designs are randomized (complete) block designs, balanced incomplete block designs, partially balanced incomplete block designs, lattice designs, etc. A block design is said to be resolvable if its blocks can be grouped so that blocks within group forms a complete replicate. Property of resolvability can be profitably used to eliminate an additional source of variation. In agricultural experiments, particularly involving large number of treatments, practical considerations dictate that the design used is resolvable. For example, large trials cannot always be completely laid out or harvested in a single session. Use of resolvable designs allows these operations to be done in stages, with one or more, complete replications dealt with at each stage. Some measurements can be made on only one or two plots of each treatment. Clearly, it is convenient to use for experimentation complete replicate formed by group of blocks (also called Superblock). Many of BIB designs are resolvable and Kageyama (1972) has given list of such design. Information on the resolvability of group divisible designs and other PBIB designs with two associate classes is given in Clatworthy (1973). Lattice designs are resolvable designs for $v=s^2$ treatments and $k = s$ plots per block and were introduced by Yates (1936). Simple lattice($r = 2$) and Triple lattice($r = 3$) are available for any integer $s>1$. Quadruple lattices are available for many but not all values of s. Yates (1940) pointed out that lattice designs due to resolvability property could never be less efficient than randomized block designs. Rectangular lattice designs are resolvable designs for $v = s\ (s-1)$ treatments and $k = s-1$ plots per block and were introduced by Harshbarger (1949). The catalogue of lattice and rectangular designs is available in Cochran and Cox (chapter 10). Further, if blocking proves ineffective, lattice designs can be analyzed by usual randomized complete block analysis.

Although series of designs mentioned above provide large number of resolvable designs, there are still large number of combinations of v and r for which no such design belonging to above classes exist or where the existing design is of low efficiency. A more general series of resolvable design for $v = ks$ treatments, called α-design has been given by Patterson and Williams (1976). For these designs there is no restriction on block sizes other than unavoidable constraint that v/k must be an integer.

2. Method of Construction

Let v be the number of treatments, then, α-Design for $v = ks$ is constructed as follows: Take kxr array say α whose elements are in the set of residue mode s. Each column of α is used to generate $(s-1)$ further columns by cyclic substitution. The resulting k x rs array called intermediate array is denoted by α^*. Finally, s is added to each element in the second row of α^*, $2s$ is added to each element in the third row and so on. The elements of the resulting array are now the symbols *0, 1, ..., v-1* representing the *v* treatments. The columns of the resulting array are the blocks of required design and each set of columns generated from the same column of α constitute a complete replication.

Example: Consider the construction of α-Design with $v = 12; k = 4; r = s = 3; b = 9$. Given the generating array α, the intermediate array α^* and hence designs are easily constructed.

Generating array α

0	0	0
0	0	2
0	2	1
0	1	1

Intermediate array α^*

0	1	2	0	1	2	0	1	2
0	1	2	0	1	2	2	0	1
0	1	2	2	0	1	1	2	0
0	1	2	1	2	0	1	2	0

Design

REPLICATE	1			2			3		
BLOCK	1	2	3	4	5	6	7	8	9
	0	1	2	0	1	2	0	1	2
	3	4	5	3	4	5	5	3	4
	6	7	8	8	6	7	7	8	6
	9	10	11	10	11	9	10	11	9

Randomization of α-Design

Allocate treatments to the numbers *1* to *v* at random, then

1. Randomize the plots within each block.
2. Randomize the blocks within each replication

Note: In some situations, especially in presence of control treatments some restriction on allotting numbers to the treatments at random are imposed.

Concurrences

The number of concurrence of two treatments is the number of blocks containing pair of treatments. In the example cited above pair of treatments *1* and *7* concur once, treatments *0* and *3* concur twice, while treatments *0* and *2* do not concur at all. A design with concurrence $g_1, g_2, \ldots, g_n$ will be referred as $\alpha\ (g_1, g_2, \ldots, g_n)$ design; the above example 1 is therefore an $\alpha(0,1,2)$ design.

Allocation of control treatments

Varietal trial includes both control varieties and entries. Three sorts of comparisons can be distinguished

(a) Between entries and controls

(b) Among entries

(c) Among controls

Now control should be allocated so that comparisons (c) which are of less interest than other comparisons are estimated with less precision. In order to achieve this, for practical purposes it suffices to arrange α-designs so that numbers of concurrences among control varieties are as small as possible thus controls should be spread out widely. This can be done by allocating first *c* varieties number in the design to control varieties, where *c* is the number of controls. Only when *c* is greater than *s* will any block have more than one control variety. In such situations randomization step (*c*) is than replaced by (*c1*) allocate control variety to numbers *1* to *c* at random and (*c2*) allocate entries numbers (*c+1*) to *v* at random. In situations where experimenter is interested in having control varieties replaced r_1 times with in each super block (replications) i.e total number of replicates of control is rr_1. This can be done by choosing a design for $V = C + v\text{-}c$ where *v* number of varieties, *c* is number of controls and $C=cr_1$. Each control variety is than given a r_1 variety code number in the design. While allocating number care should be taken that these number appear in different blocks of each super block.

Choice of designs

Patterson *et al* (1978) have given 11 basic arrays which can be used to construct α-Designs with $k <= s$ these are presented in table 1

Table 1 basic generating arrays for α - Designs

s=k=5	**s=k=10**	**s=k=6**
0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0
0 1 2 3 4	0 1 3 5 4 6 7 8 9 2	0 1 3 2 4 5
0 4 3 2 1	0 9 6 7 5 3 2 4 8 6	0 5 2 3 1 1
0 2 4 1 3	0 5 9 2 6 1 4 7 2 3	0 4 5 1 2 3
s=k=7	**s=k=8**	**s=k= 9**
0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0
0 1 2 4 3 5 6	0 1 3 5 2 4 6 7	0 1 3 7 2 4 5 6 8
0 3 6 5 2 1 4	0 2 7 3 5 1 0 6	0 8 6 2 3 1 7 5 4
0 2 4 1 6 3 5	0 6 1 4 3 6 2 5	0 7 4 3 5 6 2 1 7
s=11,k=9	**s=12,k=8**	**s=13,k=7**
0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0
0 1 4 9 2 5 6 3 7	0 1 7 9 4 11 10 5	0 1 3 9 12 8 6
0 6 8 7 3 1 5 9 4	0 2 5 6 11 3 4 1	0 4 8 2 10 5 7
0 7 1 5 6 3 10 4 1	0 3 1 4 8 10 7 6	0 10 11 1 6 12 8

s=14,k=7						
0	0	0	0	0	0	0
0	1	9	11	2	5	3
0	8	10	13	6	11	1
0	10	7	2	1	2	11

s=15,k=6					
0	0	0	0	0	0
0	1	3	7	10	14
0	8	12	2	13	3
0	7	14	5	11	8

The arrays are for four replicate designs with *k* equal to the smaller of *s* and integer part of *100/s* and *s* varying from *5* to *15*. Arrays for other values of *h>= 4* are obtained from the first *k* columns of basic array and the arrays for *r = 2* or *3* from the first two or three rows.

Unequal block sizes

Existence of α-Designs for *v* treatments in blocks of *k* plots with *k<v* implies that *v* is multiple of *k*. In many practical situations this may not be true, e.g. for *v = 10* there is no α-Designs with equal block sizes in range *3-4*. Patterson & William (1976) consider the use of two block sizes k_1 and k_2 by imposing restriction $k_2 = k_1-1$. These design exists when $v = s_1k_1+s_2k_2$ where s_1, s_2, k_1, k_2 are positive integers. These designs are derived from α-Designs as follows.

(i) Construct an α-Designs for $v+s_2$ treatments with $s=s_1+s_2$ blocks of k_1 plots in each replication.

(ii) Delete a set of s_2 treatments, no two of which concur. The varieties labelled *v*, *v+1*, . . ., $v+s_2-1$ provide such a set.

These designs have been referred as α-Designs with almost equal block sizes.

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